A New Reliability Allocation Method Based on FTA and AHP for Nuclear Power Plant

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Abstract: Reliability of Nuclear Power Plants (NPPs) has become an even greater concern in recent years. Protecting the public from the risk of NPPs by increasing the reliability is the main goal of the nuclear reliability study. In this paper, a new reliability allocation method combining Fault Tree Analysis (FTA) with Analytic Hierarchy Process (AHP) was proposed to improve extant methods, most of which were not reasonable or efficient enough to allocate reliability for complex system. With this method, objective computed result from FTA and subjective evaluation from experts was combined to make the allocation result more accurate and more reasonable.

Keywords: Reliability Allocation, Fault Tree Analysis, Analytic Hierarchy Process

1. INTRODUCTION

Reliability has become an even greater concern in recent years because high-tech industrial processes with increasing levels of sophistication comprise most engineering systems today. Based on enhancing component reliability and providing redundancy while considering the trade-off between system performance and resources, optimal reliability design that aims to determine an optimal system-level configuration has long been an important topic in reliability engineering [1].

Protecting the public from the risk of NPPs by increasing the reliability of NPPs is the main goal of the nuclear reliability study. To achieve this goal, various approaches are adopted in several aspects: (1) the diversity and redundancy concepts in the design of NPPs, (2) the surveillance and testing of components, and (3) the various safety analyses such as Probabilistic Safety Assessment (PSA). As it is impossible to keep the risk from NPPs as zero in the real world owing to various technical and economical restrictions, an approach named reliability allocation emerged. Reliability allocation is a kind of optimization problem for minimizing the total plant costs in a reasonable way and also subject to the overall plant safety goal constraints [2].

Reliability allocation was applied to determine the reliability characteristics of reactor systems, subsystems, major components and plant procedures that are consistent with a set of top-level performance goals: the core melt frequency, acute fatalities and latent fatalities. Reliability allocation can be performed to improve the design, operation and safety of a new and/or existing NPP. The importance of reliability allocation is emphasized in several aspects: (1) to minimize the risk of nuclear power plants under various constraints, (2) to allocate limited resources effectively, and (3) to reduce over-design [3].

Until now, various approaches were proposed to solve this problem. However, most approaches have some limitations in satisfying all optimization objectives. Some could reach the allocation goal efficiently, such as the Equisection method, AGREE method and Fault Tree Analysis method [4, 5]. However, they fail to take the weight and importance of each component or subsystem about different external factors into consideration, including the impact of environment, the severity of the consequences and stuff [6, 7]. On the other hand, by utilizing the expertise, some other methods

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attempt to allocate reliability based on the weight and importance of each component or subsystem considering different factors, such as the AHP, Rating distribution method and stuff. But the cost is loss of efficiency and neglect of many available accurate data [8].

So a new reliability allocation method combining FTA with AHP was proposed to improve existing methods most of which were not reasonable, accurate or efficient enough to allocate reliability for complex systems in nuclear power plant. With this method, objective computed results from FTA and subjective evaluation from experts was combined to make the allocation results more accurate and more reasonable in an efficient way.

A reliability allocation example for residual heat removal system in nuclear power plant was given to examine the validity and rationality of this method. Using this method, a computer program has been developed and the module will be available in the integrated Reliability and Probabilistic Safety Assessment program called RiskA [9-14] which was developed by FDS team [15-18].

2. Review of Literature

In this section, two typical reliability allocation methods used in this paper are reviewed. Their advantages and defects are analyzed, based on which a new reliability allocation method is necessary to be proposed.

2.1. Fault Tree Analysis

FTA is a method for identifying and documenting the combinations of lower-level subsystem events and component events that allow a top-level event (or root node) to occur. When the root node is hazard, the FTA assists in the requirements process by describing the ways in which the system can reach that unsafe state.

Fault tree analysis begins with documenting the minimum cut set of the top event. A cut set is a set of basic events whose occurrence causes the system to fail. A cut set cannot be reduced is called a minimum cut set. A minimum cut set of a fault tree gives a minimum set of events necessary to satisfy the root event. Using Fuseell-Vesely algorithm, the minimum cut-sets of a fault tree could be determined easily [19].

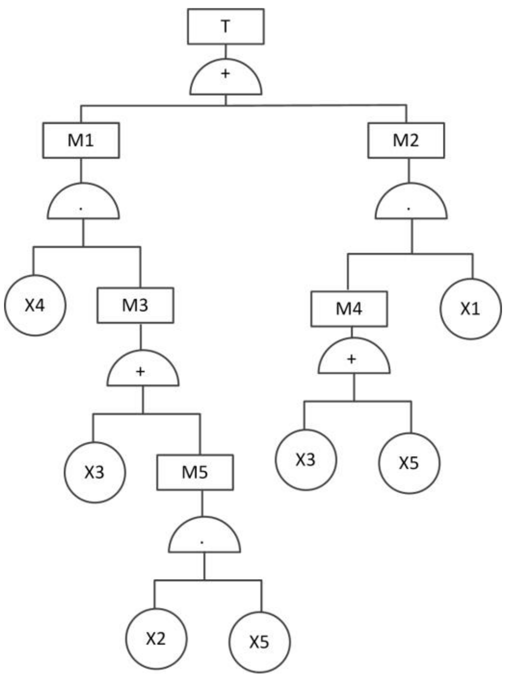

Fig. 1. Fault Tree Model of PRHR system.

A simplified fault tree model of passive residual heat removal (PRHR) system in nuclear power plant is shown in Fig. 1. Trough qualitative analysis we get four minimum cut-sets: $G_1\{X_1, X_3\}$, $G_2\{X_1, X_5\}$, $G_3\{X_3, X_4\}$, $G_4\{X_2, X_4, X_5\}$. In this paper, all cut-sets refer to the minimum cut-sets, G_i represents cut-set's name and X_i represents basic event's name.

FTA plays an important role in reliability analysis. Through quantitative analysis, we can obtain the accurate probability importance of each basic event which could indicate a sub-system or a component. In existing method, the reliability of top event, which represents the main system, could be allocated to subsystems or components by using such probability importance as a kind of weight.

Comparing with other method, FTA is a mature and fast method that could analyze and verify the logic relationship of subsystems or components and their respective reliability requirements. However, only basing on the probability importance is not enough to fulfill the rationality and effectiveness requirements of reliability allocation. Many ignored factors also could impact the allocation result obviously, such as the impact of environment, the severity of the consequences, the feasibility ant stuff. But it is not easy to obtain the accurate data of such factors. Hence, a method which could quantify subjective evaluation is needed to compensate the deficiency.

2.2. The Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) is a powerful and flexible decision making process to help people set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered [20]. An advantage of the AHP is that it is designed to handle situations in which the subjective judgments of individuals constitute an important part of the decision process. The practical nature of the AHP makes it suitable for determining the weight of each subsystem or component for reliability allocation [21].

The AHP solves the problem in four steps:

1. Setting up the hierarchical structure;
2. Collecting data by pairwise comparisons at each level;
3. Using the eigenvalue method (or other available computation methods) for computing the relative weights at each level;
4. Aggregating the relative weights of various levels in arriving at relative weights of the elements at the lower levels with respect to that at the top level of the hierarchy.

The advantages of AHP could exactly compensate the deficiency of FTA through its integration and quantification capacity for subjective views from experts which is a significant factor in NPPs. So there is a necessary to find a way combining FTA and AHP.

3. A New Reliability Allocation Method Based on FTA and AHP

A new reliability allocation method was proposed to combine FTA with AHP taking the advantage of both methods. This method includes two reliability allocation steps. Firstly, reliability allocation result of each selected minimum cut-set of fault tree is decided according to the given value of system safety target by using AHP. Secondly, on the basis of those reliability requests of each minimum cut-set, reliability allocation result of selected basic events in each cut-set can be obtained by using AHP again. Thus, a quantitative model for calculating the reliability of each component in system is set up. The procedure is started as follows.

3.1. Fault Tree Analysis

The top event of a fault tree is a whole system. Its basic events represent the component or subsystem in it. Through qualitative analysis, minimum cut-sets and basic events in each cut-set are obtained. By quantitative analysis, the probability importance of each basic event and the initial reliability R_0 of top event and each minimum cut-set are calculated. Some other data of fault tree are also needed. They are: the initial reliability of each cut-set, the initial reliability of each basic event and the mean time to failure (MTTF) of each basic event.

After FTA, we need experts to set the system reliability goal R_s based on the initial reliability of top event. Then we could get the following equations:

$$R_s \geq R_0. \qquad (1)$$

$$R_0 = \prod_{i=1}^{m} R_{\mathrm{i}} \qquad (2)$$

$$R_i = f(r_j) = 1 - \prod_{j=1}^{n} (1 - r_j) \qquad (3)$$

3.2. Allocate Reliability to Minimum Cut-sets

The first allocation step is to allocate the reliability goal as the target reliability of some selected minimum cut-sets. The procedure includes:

1. Ordering the cut-sets as $R_1 > R_2 > .. > R_m$, then choose k low reliability cut-sets as the selected cut-sets waiting to reach a new target reliability to help system meet reliability goal. Generally, the number of selected cut-sets k should be less than 9 to ensure the efficiency of calculation.

Define R_s' and R_0' as the reliability goal and the initial reliability of all selected cut-sets, and R_i' denote the reliability target of each selected cut-set. From Eq. (2), R_s' is given by:

$$R_s' = \prod_{i=1}^{k} R_i' = R_s \Big/ \prod_{i=k+1}^{m} R_i \qquad (4)$$

$$R_0' = \prod_{i=1}^{k} R_i = R_0 \Big/ \prod_{i=k+1}^{m} R_i \qquad (5)$$

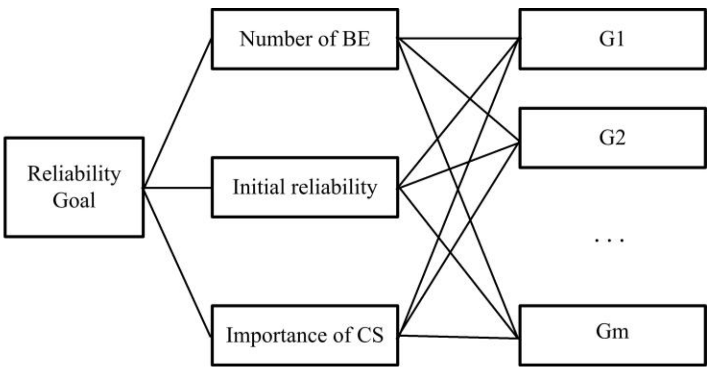

Fig.2. Hierarchical Structure of Cut-sets

2. Setting up the hierarchical structure as shown in Fig.2. The factors to be considered in criterion layer include:

The number of basic events in each cut-set: if regard cut-set as a subsystem, then the number of basic events indicates the complexity of the subsystem. Higher reliability is required if subsystem are more complex.

The reliability of each cut-set: higher reliability is required if components or subsystems are not reliable enough initially.

The importance of each cut-set: higher reliability is required if subsystems or components are more important.

3. Collecting data by pairwise comparisons, and then construct the input matrix of pairwise comparisons (IMPC) in each layer.
In this step, IMPC in criterion layer needs collecting data by pairwise comparisons from experts. The IMPC in alternative layer could be automatically filled by the data from FTA.

4. Examine the consistency of the IMPC in criterion layer. If consistency is satisfied, then go to the next step. If not, return to step 3.

5. Computing the vector of global relative weights of all selected cut-sets utilizing eigenvalue method as:

$$WC = \begin{bmatrix} \omega_c^1 \\ \omega_c^2 \\ \vdots \\ \omega_c^k \end{bmatrix} \tag{6}$$

Where $\omega_c^i, i = 1,2,\ldots,k$, represents the global relative weights of each cut-set.

6. Based on the weights of selected cut-sets, calculating their target reliability. Define:

$$R_i' = R_i + \Delta R_i \tag{7}$$

Where ΔR_i is the increment of R_i defined by ω_c^i and ΔR_i could be defined as:

$$\Delta R_1 : \Delta R_2 : \ldots : \Delta R_k = \omega_c^1 : \omega_c^2 : \ldots : \omega_c^k \tag{8}$$

Based on Eq. (7) and (8), the reliability target of the i th cut-set R_i' is given by:

$$R_i' = \frac{R_s' - R_0'}{R_0'} \cdot \frac{\omega_c^i}{\sum_{i=1}^{k} \omega_c^i / R_i} \tag{9}$$

3.3. Allocate reliability to basic events

After obtaining reliability target of each selected cut-set, the second step is to allocate the target reliability to the basic events contained by each selected cut-set. The procedure includes:

1. Choosing a selected cut-set and then ordering its basic events as $r_1 > r_2 > .. > r_n$, then choose l low reliability basic events as the selected basic events waiting to reach a new target reliability to help the cut-set meet reliability target. The number of selected basic events l should also be less than 9 to ensure the efficiency of calculation.

Define R_i'' and R_i^0 as the reliability goal and initial reliability of all selected basic events in the i th cut-set . r_i' denote the reliability target of each selected basic event. From Eq. (3), R_i'' and R_i^0 is given by:

$$R_i'' = 1 - \prod_{i=1}^{l} (1 - r_i') = 1 - \frac{(1 - R_i')}{\prod_{i=l+1}^{n} \left(1 - r_i'\right)} \tag{10}$$

$$R_i^0 = 1 - \prod_{i=1}^{l} (1 - r_i) = 1 - \frac{(1 - R_i)}{\prod_{i=l+1}^{n} (1 - r_i)} \tag{11}$$

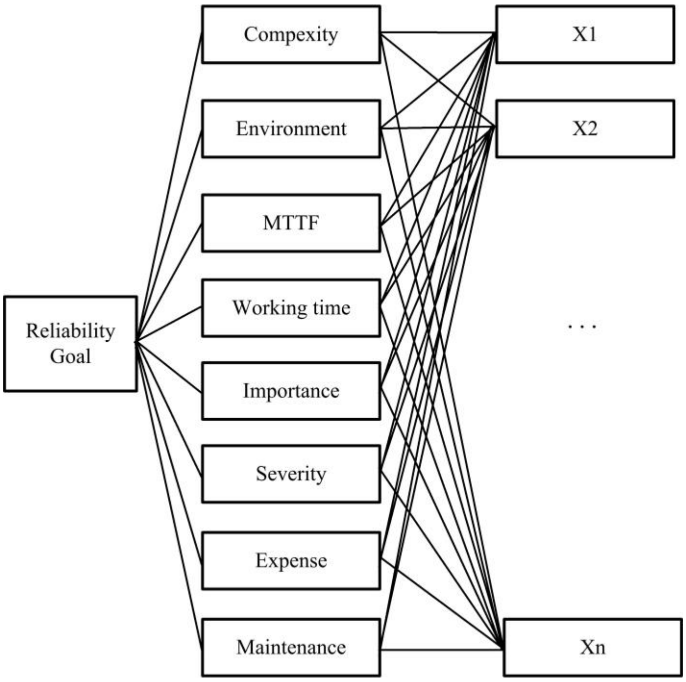

Fig.3. Hierarchical Structure of Basic Events

2. Setting up the hierarchical structure as shown in Fig.3. The factors to be considered in criterion layer include:

Complexity: the more complex the subsystem or component is, the more difficult it is to be improved. Hence, less reliability is needed of more complex subsystems or components.

Environment: lower reliability is required in a worse operating environment.

MTTF: higher reliability is required if the subsystems or components have a short MTTF.

Working time: lower reliability is required if the subsystems or components have a long working time.

Importance: Including Probabilistic importance and structural importance. Higher reliability is required if the components or subsystems are of improved importance or influence in the system.

Severity: higher reliability is required if the consequence is more severe.

Expense: higher reliability is required if the expense of raw material and spare parts is higher

Maintenance: higher reliability is required if the subsystems or components is difficult to be maintained.

3. Collecting data by pairwise comparisons, and then construct the input matrix of pairwise comparisons (IMPC) in each layer.

In this step, IMPC in criterion layer needs collecting data by pairwise comparisons from experts. Then for some factors the IMPC in alternative layer could be automatically filled by the data from FTA, such as probabilistic importance. For some other factors, such as working time and MTTF, accurate data of each subsystem or component could be found. For the rest factors, the IMPC in alternative layer could be filled by experts.

4. Examine the consistency of the IMPC in criterion layer and the IMPC filled by experts in alternative layer. If consistency is satisfied, then go to the next step. If not, return to step 3 to re-fill the IMPC which includes any inconsistency.

5. Computing the vector of global relative weights of all selected basic events utilizing eigenvalue method as:

$$WB = \begin{bmatrix} \omega_b^1 \\ \omega_b^2 \\ \vdots \\ \omega_b^l \end{bmatrix} \tag{12}$$

Where $\omega_b^i, i = 1,2,...,l$, represents the global relative weights of each basic events.

6. Based on the weights of selected basic events, calculating their reliability target. The reliability target of i th basic event r_i' could be obtained.

Based on Eq. (3), r_i' could be given by

$$1\text{-}R_i'' = \prod_{i=1}^{l}\left[1 - r_i'\right] \tag{13}$$

$$1\text{-}r_i' = \left(1 - r_i\right)^{1+s_i} \tag{14}$$

Where s_i is a parameter defined by ω_b^i and could be given by:

$$s_1 : s_2 : \ldots : s_l = \omega_b^1 : \omega_b^2 : \ldots : \omega_b^l \tag{15}$$

Based on Eq. (13), (14) and (15), the reliability target of the i th cut-set R_i' is given by:

$$r_i' = 1 - \left(1 - r_i\right)\exp\left[1 + \frac{\omega_b^i \ln\left(1\text{-}R_i''\right)/\left(1 - R_i^0\right)}{\sum_{i=1}^{l}\omega_b^i \ln\left(1 - r_i\right)}\right] \tag{16}$$

7. Then go back to step g and choose another selected cut-set to allocate its reliability target to its basic events.

3.4. Results Optimization

Based on Eq. (16), we could get reliability target of all basic events r_i' . If there is exist intersections in the cut-sets, that is to say, the result of r_i' may have p different values, let them be $r_i'^1, r_i'^2, \cdots, r_i'^p$, then we define:

$$r_i' = \max\left(r_i'^1, r_i'^2, \cdots, r_i'^p\right) \tag{17}$$

From procedure above, a quantitative model for calculating the reliability of each component in system is set up. All reliability targets of components or subsystems r_i' are obtained. To confirm the result, a case study is proposed in the next chapter.

4. Case Study

The example in this study focuses on the PRHR system in nuclear power plant, the fault tree model is shown in Fig. 1. It is used to explain the feasibility and validity of the new allocation method, and then how to determine the optimum reliability allocation and improvement of a general system.

The initial reliability of each basic event X_1, X_2, X_3, X_4, X_5 , is presumed respectively as below:

$$r_1 = r_2 = 0.98, \; r_1 = r_2 = 0.97 \;, \; r_5 = 0.75$$

Presume that X_1 is a complex subsystem in harsh environment, X_2 is a component which have severe consequence, X_3 and X_4 are components in good working environment, and X_5 represents a ordinary subsystem with low reliability.

Through quantitative and qualitative analysis of FTA, the information of cut-sets and basic events is respectively shown in Table 1 and Table 2.

Table 1: Information of Cut-sets

CS Name	Reliability	BE Number	BE Name	Importance
G_1	0.99500	2	X_1, X_3	9.03%
G_2	0.99910	2	X_1, X_5	75.29%
G_3	0.99940	2	X_3, X_4	13.55%
G_4	0.99985	3	X_2, X_4, X_5	2.26%

Table 2: Information of Basic Events

BE Name	Reliability	Probabilistic Importance	Structure Importance	CS Included
X_1	0.98	0.0533	0.4375	G_1, G_2
X_2	0.98	0.0007	0.0625	G_4
X_3	0.97	0.0489	0.4375	G_1, G_3
X_4	0.97	0.0298	0.3125	G_3, G_4
X_5	0.75	0.0199	0.3125	G_2, G_4

Where CS is short for Cut-set and BE is short for basic event.

The initial reliability of top event is obtained as R_0 =0.993359, define the reliability goal of top event as R_s =0.998000. Respectively using FTA reliability allocation method, AHP reliability allocation method and the new reliability allocation method based on FTA and AHP proposed in this paper, we could get the reliability allocation results of each method as show in Table 3.

Table 3: Information of Cut-sets

BE Name	Reliability target of FTA	Reliability target of AHP	Reliability target of the new method
X_1	0.9974	0.9885	0.9890
X_2	0.9800	0.9841	0.9839
X_3	0.9744	0.9801	0.9799
X_4	0.9754	0.9855	0.9861
X_5	0.7565	0.8233	0.8162

From Table 3, the advantages of defects of each method could be clearly realized. Firstly, FTA method increases the reliability of basic events to reach the reliability goal of top event. However, the allocation result is not reasonable enough as the reliability target of X_1 is too high to reach as a complex system in harsh environment and the reliability target of X_2 has not change although the consequence of X_2 is very severe. At the mean while, X_3 and X_4 should be allocated more reliability as they are in a good working environment.

In addition, AHP method could allocate reliability considering multiple factors and the allocation results are more rational and reasonable than FTA method for it reduce the enhancement of X_1 and increase the enhancement of X_2, X_3 and X_4. X_3 and X_4 get higher reliability for their good working environment. However, the allocation process has high dependence on expertise and it needs too much time for expert to construct the input matrix of pairwise comparisons. As a result, the reliability allocation method is not efficient enough.

Finally, the results of the method based on FTA and AHP show the same rationality as AHP method, it even improve the results based on importance. Although considering more factors than AHP method, the allocation process is efficient enough for the application of quantitative analysis from FTA.

4. CONCLUSION

In this paper, a new reliability allocation model is established to ensure the rationality and efficiency as it is a significant part of the planning and design stages of complex systems in nuclear power plant.

The model in this paper is an amalgamation of qualitative and quantitative information reflecting the subjective views and objective facts. The expert's views about the nuclear power plant with objective calculation results from fault tree are integrated. A hierarchical structure is used to take more factors such as complexity, severity and environment into consideration, then applied AHP to arrive at the relative importance (or global weights) of subsystems and components that embody the structures of the hierarchy. Utilizing allocation functions, final results that are confirmed in case study are got.

The method has following advantages:
1) It takes advantages of FTA and AHP combining subjective view with objective facts to ensure the efficiency and rationality of reliability allocation.
2) It is simple and efficient because we can stay away from the difficulty of non-linear calculating.

The proposed allocation model is part of RiskA. RiskA is designed to assist in reliability and risk analysis. Therefore, the proposed model can be easily applied. Now, a module using this method has been developed and is available in RiskA.

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