A stochastic production planning optimization for multi parallel machine under leasing Contract

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Abstract: In this paper, we aim at optimizing the production planning. The problem consists on a several identical machines mounted in parallel and which are leased depending on a fluctuating demand over a finite time horizon under given service level. The objective of the production plan is to determine the best combination of leased machines numbers, production time (or level) and inventory levels, by developing a mathematical model, that minimize the average total costs over the finite time horizon. The contribution and newness of this work is that it treats this approach under new constraints related especially to leasing techniques and consequently we assume that the number of workstations varies from a production period to another. This characteristic is due to the leasing principle as well as to the fluctuating demand that we have to take into account. A numerical example confirms the analytical study.

Keywords: Optimization, Production planning, Leasing constraint, Random demand, Service level, Multi parallel machine.

1. INTRODUCTION

Ameliorating the industry situation requires certainly good making decisions and good management. These decisions can be achieved in order to improve the product quality, to reduce the costs and maximize the customer service level. In this context, we can consider the leasing as a very important solution for many manufacturing to minimize costs and guard against important competitiveness. In order to reduce the manufacturing cost, more industries have preferred leasing, newer and better equipment appears on the market and that these owning costs is increasing, rather than owning it. In this context, we can cite the work of [6] where based on the causes that newer equipment appears on the market and that the cost of owning the equipment is increasing, more businesses have started leasing equipment rather than owning it [1]. [1] considered a business model in which a company leases new equipment and sells the remanufactured one at the same time. They developed a dynamic program formulation to determine the optimal price of the remanufactured equipment and the optimal payment structure for the leased equipment to maximize the profit. [4] deal with the choice between purchase and lease in the context of radiotherapy equipment.

In the other hand, the optimization of production which minimizes the total production and inventory cost is one of the principal activities of a hierarchical decision manufacture process. In the context of production/inventory optimization problem, [2] proposed a model which defines an inter-temporal quadratic cost minimization program and that by approximating the cost functions for hiring labour and lay-off, overtime, inventory and shortage by suitable quadratic functions. As a result, and considering some constraints, this model provides an optimal smoothing solution for aggregate inventory, production and workforce. The HMMS approach has been extensively used and a source of inspiration in literature [7]. It is usually applied as a benchmarking tool in order to compare different production planning approaches and to provide managers and decision makers with perspectives and ideas about how to manage the firm's material resources. But, some other works such as [3] proved that this quadratic approach is useful to evaluate the production process. So, for example, the quadratic inventory cost describes and takes into account both possible status of inventory: negative (rupture and backorders) and positive (overstocking). [9] established a model in three phases. The first one consists in considering products requiring the same resources to find an aggregate production plan. In this phase, the objective is to minimize the costs of set up, production, inventory, workforce and maintenance. The second step is to determine a director production plan minimizing the disparity between a normal production plan and the aggregate plan. The last step consists in simulating the workstations breakdowns.

Recently, more works, concerning the production optimization, have integrated new constraints. The model of [5] is based on the control aspect of the production of a system, which is composed of *m* identical machines able to produce *n* types of products and subjected to recuperation periods after a troubleshooting. The decision variables in this research are the production rate and the repairing date. The main objective of the work is to minimize the expression of the total cost under these two variables.

Inspired from the HMMS model, we had the idea to make emphasis on the machines instead of workers, production rate and inventory levels in order to make the best and optimal production planning. Also, in our work, we make some changes on the model keeping its linear quadratic form. Furthermore, we take into account some constraints on the decision variables to make our approach more realistic and to ensure its applicability in real industrial cases.

The objective of this paper is to determine the economical production planning taken into account leasing machines number and random demand to minimize the sum of production, inventory and leasing costs.
This remainder of this paper is organized as follows: Section 2 proposes a general stochastic production, inventory model. Section 3 presents the problem formulation with the total cost expression as the diverse Costs of production system. Section 4 presents and develops the policy and analytical expression of production (works) and inventory. A simple numerical example is presented in section 5. Finally, the conclusion is given in Section 6.

2. PROPOSED MODEL

2.1. Notations

We used the following notations in this paper:

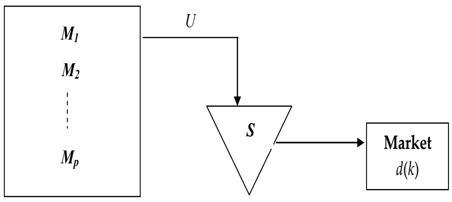
Δt : length of a production period
H : number of production periods k (k=0, 1,…, H).
$H.\ \Delta t$: finite time horizon.
$\hat{d}_k$: average demand during period k (k=0, 1,…, H).
V_{d_k} : variance of demand during period k (k=0, 1,…, H).
M_k : number of leasing workstations during period k.
U_k : production rate during period k (k=0, 1,…, H).
u_{ik} : quantity produced by machine i in period k.
S : inventory level at the end of period k (k=0, 1,…, H).
M : maximum number of leasing machines per period.
m : minimum number of leasing machines number per period.
u : production rate of a machine per time unit (hour); (the same for all machines).
X_{ik}^N : number of time units (hours) that machine i works in period k (Normal time).
X_{ik}^S : number of time units (hours) that machine i works in period k (Over time).
$X_{\max}^N$: maximum of time units that a machine could perform during a period and in normal time.
$X_{\max}^N$: maximum of time units that a machine could perform during a period and as overtime.
c_1 : Cost of a machine leasing;
c_2 : fixed costs of leasing;
c_3 : Cost of variation in leasing machines number;
c_4 : Coefficient describing asymmetry between increasing / decreasing machines number;

c_5 : Cost of overtime work (per time unit);
c_6 : Cost of a time unit not used in normal time;
c_8 : Cost of storage/rupture (per unit);

2.2. Problem description

In this work, we consider a problem of production optimization planning and that under new approach considering a leasing constraint. The manufacturing system is composed of a several identical leasing machines, mounted in parallel, which produces one type of product in order to meet a random demand and minimizes the total production/inventory/leasing cost. We can consider two categories of the work time of equipment's. First category: where each machine works normally during maximum time units X_{ik}^{N} per period. Second category: in order to satisfy the random demand that exceeds the production, it's preferable to make machines work in overtime which not exceeds a maximum time units X_{ik}^{S}. The number of leasing machines varies from period to another depending to the production quantity and the random demand and consequently it represents decision variables to be determined under some constraints. We assume that the production horizon *H* is divided into equal period's *Δt*. The customer demand which is random and given by a normal distribution. See figure1

Figure 1: Problem Description



Our problem consists in determining the best combination of leased machines numbers, production time or production rates and inventory levels under service level and leasing constraints. That minimizes the average total costs over the finite time horizon in order to satisfy the fluctuating demand.

3. PROBLEM FORMULATION

3.1. Total production cost

The idea is to minimize the expected costs of leasing machines numbers, production rates and inventory levels take into account some constraints on main variables. This kind of problem can be formulated as a stochastic quadratic optimization problem under a stock threshold level constraint, with numbers of leasing machines, normal time unit of machine work and overtime unit of machine work $(M_k^*, X_{ik}^N, X_{ik}^S)$ corresponding to each period as the decision variables and which is equivalent also to (M_k^*, U_k^*).

The stochastic problem as follows:

$$\underset{(M_k,U)}{Min}\left(\sum_{k=1}^{H} E\left\{\begin{bmatrix}\left(c_1\cdot M_k+c_2\right)+\left(c_3\cdot\left(M_k-M_{k-1}-c_4\right)^2\right)+\left(c_8\cdot\left(S_k-\left(a_1+a_2\cdot d_k\right)\right)^2\right)\\ +c_5\cdot\theta_t\cdot\Delta t\cdot\sum_{i=1}^{M_k}X_{ik}^S+\left(1-\theta_k\right)\cdot c_6\cdot\Delta t\cdot\sum_{i=1}^{M_k}\max\left[0,\left(X_{\max}^N-X_{ik}^N\right)\right]\end{bmatrix}\right\}\right)$$

Subject to:

$$U_k = u \times \Delta t \times \sum_{i=1}^{M_K} \left(X_{ik}^N + X_{ik}^S \right) \tag{1}$$

$$S_k = S_{k-1} + U_k - d_k \tag{2}$$

$$\text{Prob}\left(S_k \geq 0\right) \geq \boldsymbol{\beta} \tag{3}$$

$$m \leq M_k \leq M \tag{4}$$

$$0 \leq X_{ik}^N \leq X_{\max}^N \tag{5}$$

$$0 \leq X_{ik}^S \leq X_{\max}^S \tag{6}$$

The first constraint determine the production rate of each period which calculated according to the time of each machines works (normal and overtime) and the production rate of each machines per time unit (hour) u. According to first constraint, determining the decision variable U_k is equivalent to determine X_{ik}^N and X_{ik}^S . Constraint (2) describe the inventory level of the store S at each production period, is formulated in the form of flow balance constraints where the inventory level of S at the period k equals to the inventory level at period k-1 plus the production rate during period k, minus the demand during period k. The service level requirement constraint is determined by the probability constraint on the stock level at each period k expressed by the constraint (3). The probability variable β which can be interpreted as the degree between the high and low customer service level. Constraint (4) defines the maximum and minimum number of leasing machines of each period. Similarly for constraints (5) and (6) that gives the assumption on normal and over time work variables.

3.2. Costs of works time (production), Inventories and leasing machines

Using the model carried out by [2], the purpose of this subsection is to develop the expected works time (production), inventories and leasing machines costs over the finite time horizon H.

- Cost of leasing machines:

$$c_1 \cdot M_k + c_2 \tag{7}$$

Where c_1 is the average cost for leasing a machine and c_2 represented the fixed cost term in the leased contract.

- Cost of variation in machines number:

$$c_3 \cdot \left(M_k - M_{k-1} - c_4\right)^2 \tag{8}$$

This equation defines the costs due to variation in the number of the machines required for period k. Constant term c_4 presents the asymmetry in costs of adding and eliminating machines.

- Costs of overtime work and unexploited capacities:

These costs expressions can be defined as follows:

- Cost of overtime work:

$$c_5 \cdot \theta_k \cdot \Delta t \cdot \sum_{i=1}^{M_k} X_{ik}^S \tag{9}$$

These costs depend to binary variable θ_k. We can noted that θ_k is equal to 1 if the random demand of each period k exceeds the production quantity when all used machines M_k works during their maximum normal time and equal to 0 otherwise.

- Cost of unexploited capacities:

Other hand, if the demand is not very important and all or some machines could work less than their maximum normal work time ($\theta_k=0$). In this case, we can consider as if those machines were over paid. This cost is given as follows

$$\left(1-\theta_k\right)\cdot c_6 \cdot \Delta t \cdot \sum_{i=1}^{M_k} \max\left[0, \left(X_{\max}^N - X_{ik}^N\right)\right] \tag{10}$$

- Inventory Costs:
  Using HMMS model, we can define the inventory cost

$$c_8 \cdot \left(S_k - \left(a_1 + a_2 \cdot d_k\right)^2\right) \tag{11}$$

with an optimal level of net inventories is equal to:

$$a_1 + a_2 \cdot d_k$$

4. ANALYTICAL STUDY

4.1. Deterministic problem

Due to the stochastic nature of our problem, the constraints and the dimensionality, to try to obtain an optimal solution can become a hard task. An approach that transforms the stochastic problem into a deterministic equivalent is necessary. This deterministic problem maintains the main properties of the original problem. Also, such a technique has the advantage of giving the possibility of exploiting different mathematical programming methods in order to solve the equivalent obtained model.

The principle step of an equivalent deterministic problem for the above stochastic one may be obtained by setting the variables equal to their means values. In this context, we can cite the works of [8] which uses the linearity of the model and assumes that the demand random variation can be described as a Gaussian process.

Before proceeding, the following notation is introduced:

Mean variables:

$E\left\{S_{i,k}\right\} = \hat{S}_{i,k}$

$E\{U_k\} = U_k$

$E\{M_k\} = M_k$

$E\{X_{ik}^N\} = X_{ik}^N$

$E\{X_{ik}^S\} = X_{ik}^S$

Variance variables: $V_{U_k} = 0$, $V_{M_k} = 0$, $V_{X_{ik}^N} = 0$, $V_{X_{ik}^S} = 0$. (Note that this reflects the fact that the control variables U_k, M_k, X_{ik}^N and X_{ik}^S are deterministic).

Using the above notations, the first constraint describing the stock level can be written under a deterministic form as follows:

$$\hat{S}_k = \hat{S}_{k-1} + U_k - \hat{d}_k \tag{12}$$

- Service level constraint

Generally in stochastic cases, it is complicated just to guarantee feasibility, though one possibility of overcoming such difficulty is to consider probabilistic constraints. Another important transformation changes the service level constraint into equivalent, but deterministic inequalities by specifying, through the following lemma, a minimum cumulative production quantity depending on the service level requirements.

Lemma 1:
We recall that β defines the targeted service level as expressed by constraint (3), repeated below:

$$\text{Prob}\left[S_k \geq 0\right] \geq \beta$$

Then, for $k=1,..,H$ we have:

$$\Pr ob\left(S_k \geq 0\right) \geq \beta \;\; \Rightarrow \;\; \left(U_k \geq \left(V_{d(k)}\right) \times \varphi^{-1}\left(\beta\right) - S_{k-1} + \hat{d}_k\right) \qquad k=1,....,H \tag{13}$$

φ : Cumulative Gaussian distribution function with mean $\hat{d}_k$ and finite variance $V_{d(k)}$

φ^{-1} : Inverse distribution function

Using the above notations, the formulation of our planning problem becomes as follows:

$$\underset{(M_k,U,N)}{Min}\left(\sum_{k=1}^{H}\left\{\begin{array}{l}\left(c_1 \cdot M_k + c_2\right) + \left(c_3 \cdot \left(M_k - M_{k-1} - c_4\right)^2\right) + c_5 \cdot \theta_t \cdot \Delta t \cdot \sum_{i=1}^{M_k} X_{ik}^S \\ + \left(1-\theta_k\right) \cdot c_6 \cdot \Delta t \cdot \sum_{i=1}^{M_k} \max\left[0, \left(X_{\max}^N - X_{ik}^N\right)\right] + c_8 \cdot \left(\hat{S}_k - \left(a_1 + a_2 \cdot \hat{d}_k\right)\right)^2\end{array}\right\} \\ + \left(1+a_2\right)^2 \cdot V_d \cdot \frac{H}{2} \cdot \left(H+1\right) + a_2^2 \cdot V_d \cdot \frac{H}{2} \cdot \left(H-1\right)\right)$$

Subject to:

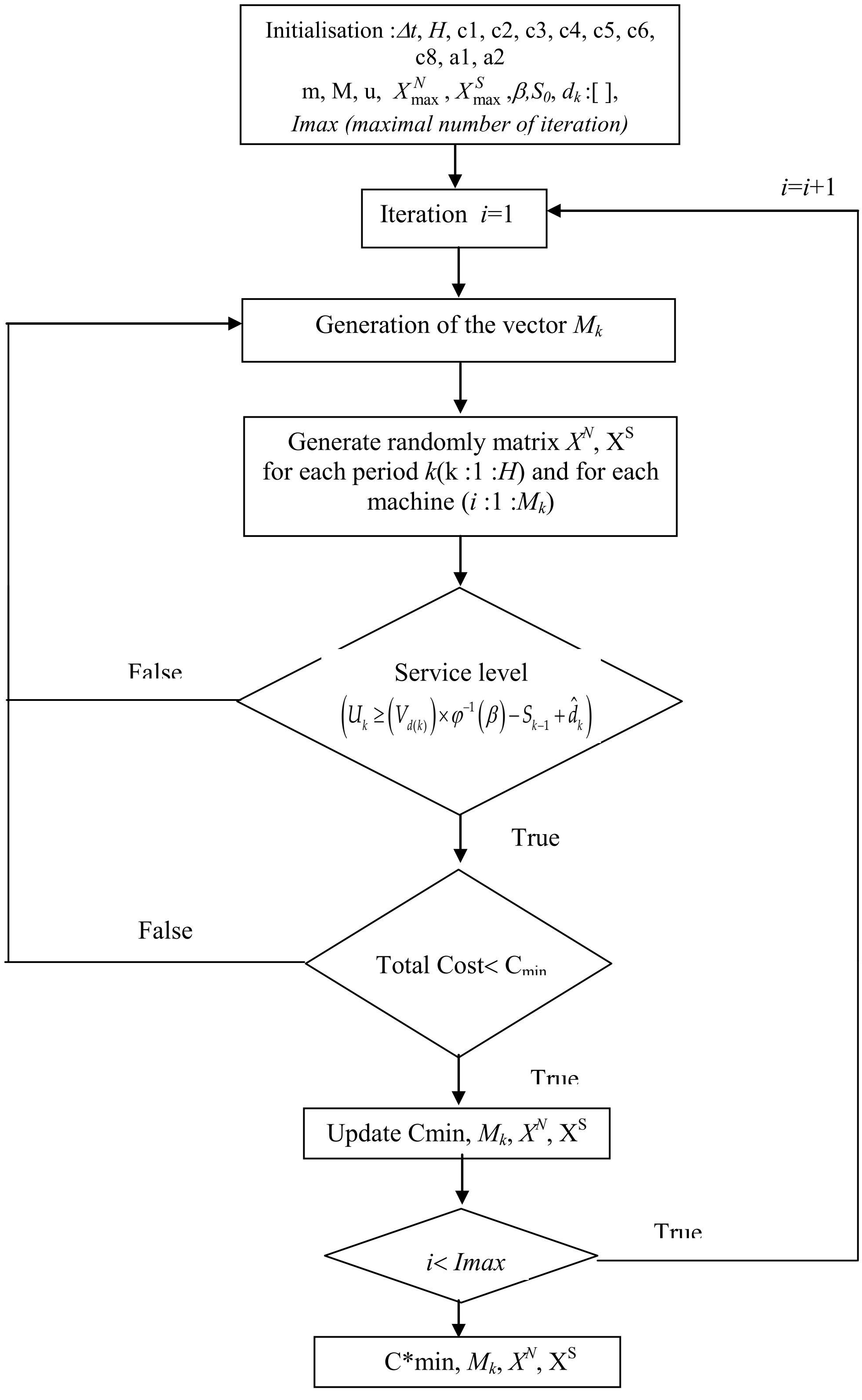
$$\hat{S}_k = \hat{S}_{k-1} + U_k - \hat{d}_k$$
$$\left(U_k \geq \left(V_{d(k)}\right) \times \varphi^{-1}\left(\beta\right) - S_{k-1} + \hat{d}_k\right)$$
$$m \leq M_k \leq M$$
$$0 \leq X_{ik}^N \leq X_{\max}^N$$
$$0 \leq X_{ik}^S \leq X_{\max}^S$$

4.1. Numerical Resolution Method

Regarding the complexity of our problem, the analytical resolution remains difficult and finding an exact optimal solution remains difficult. That is why; we proposed an algorithm or a numerical procedure and a method in order to determine an approximate possible solution. Among the difficulties are the random demand and the dependence between the decision variables. The objective of this algorithm is to determine the number of machines M_k for each period and then determine their work time's vectors together with the production rate of each period k.
The principle of this algorithm is based on Branch and Bound algorithm. It is used to generate randomly the vector of number of machines during the finite horizon and then generate randomly also the vectors of work times (normal and extra time). This generation necessitate the used of smaller intervals by reducing the first interval [m,M] and dividing it into two major smaller intervals.

The following chart illustrates the syntax of the principal algorithm that we implement on Matlab in order to find approximate solutions:

Figure 2: Problem Description



5. NUMERICAL EXAMPLE

Let us consider a system that produces one type of products to meet the random demands below. Using the models described in previous sections, we will determine the optimal production plan minimizing the total cost over a finite planning horizon: H=5 periods each of Δt=25 days duration. We supposed that the standard deviation of demand of product is the same for all periods $\sigma_d=1.1$ and the initial inventory level we assume that S_0=0.

The data required to run this model are given in sequence.
For the cost coefficients c_i and the constants a_i :
c_1=500 ; c_2=0; c_3=3.88 ; c_4=2.42; c_5=7 ; c_6=5; c_8=10 ; a_1=250; a_2=0 ;
The number of machines as well as the work time hours should obey to the following bounders:
m=6 machines; M=12 machines; u=15; $X^N_{\max}$=6; $X^S_{\max}$=2;
The customer satisfaction degree, associated with the stock constraint, is equal to 90% (β=0.9).
The average demand is presented in tables 1 below:

Table 1: Average Demands

$\hat{d}_1$	$\hat{d}_2$	$\hat{d}_3$	$\hat{d}_4$	$\hat{d}_5$
2286	22853	11880	15123	20653

We used the numerical procedure method with Matlab, in order to realize this optimization. The economically production plan and leasing machines number, the normal work time planning and an optimal overtime work scheduling are presented respectively in table 2, 3,4 with a minimal total cost equal to 1, 27108 (Monetary Unit).

Table 2: Economical leasing machines, inventory and production plans

Periods k	k=1	k=2	k=3	k=4	k=5
M_k	7	8	7	7	8
S_k	8990	4991	5129	4378	3888
U_k	11250	18250	12750	12750	19500

Table 3: Normal work time planning of each leasing machine

Periods k	k=1	k=2	k=3	k=4	k=5
Machine 1	4	6	4	4	6
Machine 2	4	4	5	4	5
Machine 3	4	6	4	6	6
Machine 4	4	4	5	5	4
Machine 5	4	4	4	4	6
Machine 6	5	5	4	6	5
Machine 7	5	6	4	6	5
Machine 8	0	6	0	0	6
Machine 9	0	0	0	0	0
Machine 10	0	0	0	0	0
Machine 11	0	0	0	0	0
Machine 12	0	0	0	0	0

Table 4: Optimal overtime work planning of each leasing machine

Periods *k*	*k*=1	*k*=2	*k*=3	*k*=4	*k*=5
Machine 1	0	0	0	0	2
Machine 2	0	0	0	0	0
Machine 3	0	2	0	0	2
Machine 4	0	0	0	0	0
Machine 5	0	0	0	0	2
Machine 6	0	2	0	2	0
Machine 7	0	2	0	2	0
Machine 8	0	0	0	0	0
Machine 9	0	0	0	0	0
Machine 10	0	0	0	0	0
Machine 11	0	0	0	0	0
Machine 12	0	0	0	0	0

According the tables, we remark that on the normal and over time planning that the machines not working in the normal time for all periods; we cannot be used in the overtime. Other, some machines work less than their maximal normal work time that generates other cost but can be an advantage to optimize maintenance cost for these machines. Other hand, we remark that the stock levels relatively important. That is related to the service rate constraint which is important.

4. CONCLUSION

This paper dealt new approach consisting in optimizing stochastic production planning problem for a several leased machines considering a random demand and a service level. The contribution and originality of this study is that it treats this approach under new constraints related especially to leasing techniques by considering the variation of machines number from production period to another in order to satisfy the random demand and minimize the total production, inventory and divers cost. Given a service level, we have formulated and solved the related stochastic production problem. An optimization has been performed obtaining an optimal production plan as well as the best combination of the leased machines number and the time work of each one over the different production periods.

In order to go ahead with this work and ameliorate it, many extensions may be recommended. In fact, we can assume a maintenance strategy of the different leased machines taking into account the influence of the production rates on the degradation degree of each machines and consequently on the preventive maintenance plan.

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