Expected maintenance costs model for time-delayed technical systems in various reliability structures

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Abstract: In the article, there is presented the mathematical model of expected maintenance costs of two- and multi-unit system in a single cycle of operation (between (*i*-1)th and *i*th time moments of inspection action performance), provided that at the beginning of the inspection cycle system elements are in the same age and show no signs of forthcoming failure. The mathematical modelling of maintenance decisions for such a system is provided with the use of delay-time analysis. Moreover, there was examined the compatibility of developed analytical model with a simulation model. The directions for further research work are defined.

Keywords: delay-time modelling, multi-unit systems, analytical model, simulation.

1. INTRODUCTION

As machine become more complex, the possibility that failures or deterioration of machine systems can result in economic losses increases. Much importance has been, therefore, attached in maintenance to reduce such losses.

One of the main functions of maintenance is to control the condition of facilities. A technique called delay time analysis has been developed for modeling the consequences of an inspection policy for any systems [8]. The central concept here is a delay time h, of a fault, which is the time elapsed from when a fail could first be noticed until the time, when its repair can be delayed no longer because of consequences (see e.g. [6, 7]) This concept, which provide useful means of modelling the effect of periodic inspections on the failure rate of repairable technical systems, was developed by Christer et al., see e.g. [8, 11, 12, 14, 31].

Known in the literature technical systems maintenance models which base on delay-time concept implementation, can be divided into two main groups [34]:

- inspection models for single-unit or complex systems;
- inspection models for multi-unit systems.

The maintenance modelling issues for single-unit and complex systems have been extensively analyzed in the literature see e.g. [20]. In the case of a repairable component, it is possible to model the reliability, operating cost and availability functions when pdf of delay time $f_h(h)$ and pdf of initial point u $g(u)$ are known. In the case of multi-component or complex system, the arrival pattern of defects within the system is modelled by an instantaneous arrival rate parameter $\lambda(u)$ at time u. If $\lambda(u)$ is constant, the model is a Homogeneous Poisson Process type (HPP), otherwise it is of a Non-Homogeneous Poisson Process type (NHPP) [27].

The inspection models for multi-component systems are widely known in the literature. The basic multi-component system delay-time model is given in e.g. [9, 13, 14, 35]. The basic assumptions include perfect inspection case, independent system components, the number of defects arising over T following a HPP with a constant rate λ per unit time, known pdf $f_h(h)$, regular inspection actions performance which requires a constant time d. Following these assumptions, probability of defect arising as a failure $P_b(T)$ and downtime per unit time $E_d(T)$ may be estimated. The basic extensions for analyzed delay time model regard to the non-perfect inspection case (see e.g. [4, 32, 33]), or multiple nested inspections case (see e.g. [36, 31]).

Moreover, in the literature there can be also found multi-component inspection models, which assume that the defect arrival process is non-homogeneous (see e.g. [7, 15, 16, 35]). Aspects of testing for trend and fitting a NHPP process to data are discussed e.g. in [3]. Moreover, the imperfect inspections case is analyzed in e.g. [4, 32].

Another important aspect which cannot be neglected is the problem of delay time model parameters estimation. This research issues are investigated e.g. in [7, 33, 34, 38].
The presented above maintenance models regard to PM policy consideration. The implementation of condition monitoring policy in delay-time models for complex systems is given e.g. in [5, 18, 37].
Many works have been carried out on the DT modelling to production plants (e.g. [1, 26]). Other works include the application to gearbox failure process (see e.g. [28]), modelling PM for a vehicle fleet (see e.g. [12, 17, 19, 30]), PM process for a coal face machinery [5], wind turbine maintenance optimization (see e.g. [2]), modelling maintenance of fishing vessel equipment (see e.g. [29]), or medical equipment (see e.g. [10]).

To sum up, most of the known in literature maintenance models regard to complex or multi-unit systems performance optimization issues. However, when assuming that the working elements are in non-series reliability structure, the analytical delay-time models are almost not investigated. Following this, authors focus on the development of analytical maintenance model of technical systems performing in various reliability structures. Thus, in the next Section, there is presented the investigated maintenance model for two-element and *n*-element system. Following this, in the next step, authors examine the compatibility of developed analytical model with simulation results, obtained from the model presented e.g. in [23]. Based on this research, it was possible to analyze the possibility of the presented model use for time between inspections period optimization issues. The work ends up with summary and directions for further research.
In conclusion, this article is to be the continuation of consideration about mathematical modelling of technical objects with time delay maintenance, presented e.g. in [20]. Moreover, it is also a continuation of the considerations about future research, connected with delay time modelling for complex and multi-unit systems in given reliability structure, developed e.g. in [21, 23, 24, 25, 39, 40].

2. DELAY-TIME MODEL FOR MULTI-UNIT SYSTEMS

In this Section, first there is considered a basic delay-time model for two-unit systems. It is assumed, that system elements working independently under the same conditions. Moreover, components are prone to become defective independently of each other when the system is in operating. In the second step of analytical model formulation, there is made an assumption that system is comprised of n elements. Moreover, there is investigated system elements performance in three main reliability structures – series, parallel and k-out-of-n.
The performed maintenance policy bases on Block Inspection policy (*BI*) which assumes, that inspections take place at regular time intervals of T, and each requires a constant time. The inspections are assumed to be perfect. Thus, any component's defect, which occurred in the system till the moment of inspection, will be identified. All elements with identified defects will be replaced within the inspection period.
In the analyzed system may be performed one of the two maintenance operations: failure repair or inspection together with replacement of elements with defects. Following this, it is assumed that when a system failure occurs, there is only performed replacement of failed components without additional inspection action performance. However, in the case of planned inspection action performance, the replacement will be performed only for those elements with visible symptoms of their future damage.
The performance of the investigated system is also defined by the additional assumptions:

- the system is a three state system where, over its service life, it can be either operating, operating acceptably or down for necessary repair or planned maintenance,
- failures of the system are identified immediately, and repairs or replacements are made as soon as possible,
- inspection action performance begins the new maintenance cycle for the analyzed system,
- maintenance actions restores system to as good as new condition,
- the system can remain functioning in an acceptable manner until breakdown (despite having defects),
- system incurs maintenance costs of: new elements, when they are replaced c_r, inspection costs c_i, and some additional, consequence costs, when system fails c_f;

- time of defects occurrence defined by u is a random variable described by its probability distribution functions $g(u)$ and $G(u)$,
- the length of the delay time before element's failure is random and its probability distributions are given by $f_h(h)$ and $F_h(h)$.

Moreover, there should be underlined here, that the developed mathematical model gives the possibility for estimation of expected maintenance costs for system, which elements are as good as new at the beginning of the maintenance cycle (e.g. first maintenance cycle performance). The maintenance cycle is here understood as the time between the two consecutive inspection actions performance.

2.1. Reliability models of system with delay time working in various reliability structures

When system performs in a series reliability structure, there is a possibility to define the *C.d.f.* of time to failure, $F(x)$, as the convolution of u and h such that $u + h \leq x$:

$$F(x) = 1 - \left[1 - \int_{u=0}^{x} g_1(u) F_{h1}(x-u)du\right] \cdot \left[1 - \int_{u=0}^{x} g_2(u) F_{h2}(x-u)du\right] \tag{1}$$

And the system reliability function: $R(x) = 1 - F(x)$.

The formulae (1) for multi-unit systems may be defined as:

$$F(x) = 1 - \prod_{j=1}^{n}\left[1 - \int_{u=0}^{x} g_j(u) F_{hj}(x-u)du\right] \tag{2}$$

When the system elements work in a parallel reliability structure, the *C.d.f.* of time to failure, $F(x)$, given by the formulae (1) may be estimated as:

$$F(x) = \int_{u=0}^{x} g_1(u) F_{h1}(x-u)du \cdot \int_{u=0}^{x} g_2(u) F_{h2}(x-u)\,du \tag{3}$$

and respectively for multi-unit systems as:

$$F(x) = \prod_{j=1}^{n} \int_{u=0}^{x} g_j(u) F_{hj}(x-u)du \tag{4}$$

Taking into account n-element system, when its elements work in the most general, k-out-of-n reliability structure, the system reliability function may be estimated as:

$$R(x) = \sum_{l=1}^{m} R^{l}(x) \tag{5}$$

where:
$R^l(x)$ – probability of system correct operation related to lth combination of up-stated elements providing the system being up state
m – number of possible combinations of up-stated elements which provide the up-state of the system (number of system up states)

And the system *C.d.f.* of time to failure: $F(x) = 1 - R(x)$.

The probability of system correct operation for lth combination of system elements being in up-state in order to system up-state providing, may be estimated as:

$$R^{l}(x) = \prod_{j=1}^{n}\left[R_j(x)\right]^{e_j}\left[1 - R_j(x)\right]^{(1-e_j)} \tag{6}$$

where:
e_j – indicator defined as follows:

$$e_j = \begin{cases} 1, & \text{if } j\text{th element in } l\text{th combination is in up state} \\ 0, & \text{if } j\text{th element in } l\text{th combination is failed} \end{cases} \quad (7)$$

$R_j(x)$ – element's reliability function, given by the formulae:

$$R_j(x) = 1 - F_j(x) = 1 - \int_{u=0}^{x} g_j(u) F_{hj}(x-u) du \quad (8)$$

As a result, formulae (6) can be expressed as:

$$R^l(x) = \prod_{j=1}^{n} \left[1 - \int_{u=0}^{x} g_j(u) F_{hj}(x-u) du\right]^{e_j} \left[\int_{u=0}^{x} g_j(u) F_{hj}(x-u) du\right]^{(1-e_j)} \quad (9)$$

2.2. Maintenance model for system with delay time

The expected costs of two-element system maintenance in one inspection cycle are defined as:

$$C(T_i) = C(t_{i-1}, t_i) = \frac{c_f F(t_i) + c_r(1-F(t_i))(G_1(t_i) + G_2(t_i)) + c_i(1-F(t_i))}{(1-G_1(t_{i-1}))(1-G_2(t_{i-1}))} \quad (10)$$

The maintenance cost expressed in the equation (10) presents the sum of possible cost: of a system failure, replacement cost of working elements with observable defects and inspection costs per a single inspection period. The formulae may be used under the condition that all the system elements were as good as new at the beginning of the cycle. The model may be developed to the form usable for multi-unit systems:

$$C(T_i) = C(t_{i-1}, t_i) = \frac{c_f F(t_i) + c_r(1-F(t_i))\left(\sum_{j=1}^{n} G_j(t_i)\right) + c_i(1-F(t_i))}{\prod_{j=1}^{n}\left(1-G_j(t_{i-1})\right)} \quad (11)$$

3. CONVERGENCE OF THE MODEL WITH LITERATURE FINDINGS

The majority of literature findings deal with results of simulation maintenance models of multi – unit system with delay time. In order to present the convergence of the foregoing model and simulation ones, presented in the literature, authors focus on the general case for system in *k-out-of-n* reliability structure. The analysis results for other reliability structures were given in [22].

The cost results for chosen case of *k-out-of-n* system are depicted in the figures 1-4. The costs are presented in the function of the length of inspection period (*T*). The *Monte Carlo* simulation was conducted for the first inspection cycle to realize assumption regarding all new elements at its beginning. The table 1 presents system parameters assumed in the simulation model.

Table 1: Simulated system parameters

Notation	Description	Value
k out of n	Reliability structure of a system	*2 out of 3*
c_e	The unit cost of new element	5
c_i	The unit cost of inspection	1
c_c	The unit cost of consequences pertaining a system failure	100
$G(u)$	Probability function of *u* period (the time that elapses from the moment when a defect arises in a new element)	$G(u) = 1 - e^{-(u/75)^{3,5}}$
$F_h(h)$	Probability function of *h* period (the time between first signals of defect and element's failure)	$F_h(h) = 1 - e^{-(h/35)^{3,5}}$

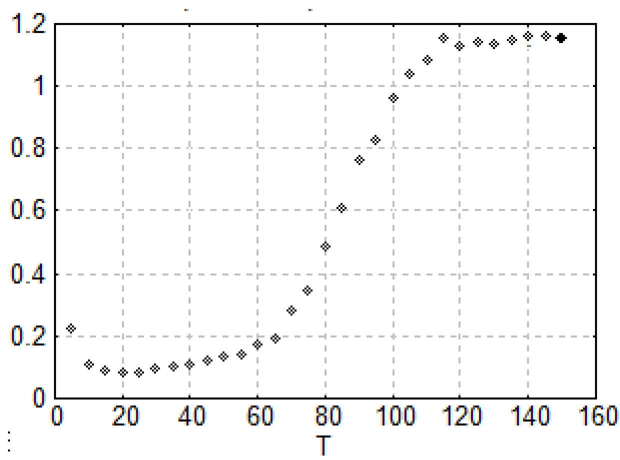

Fig. 1 The simulated expected maintenance costs of *2-out-of-3* system in a single inspection cycle

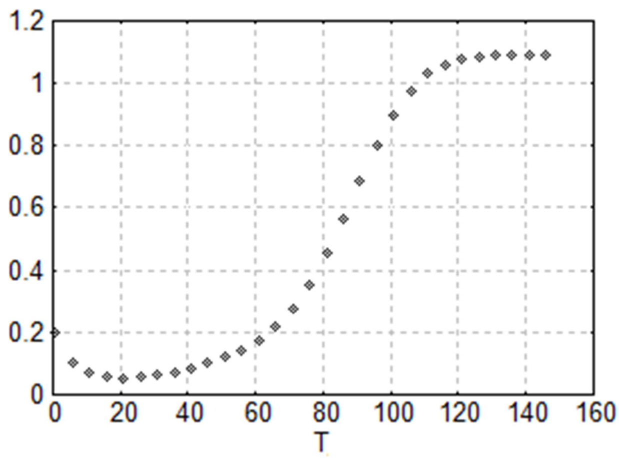

Fig. 2 The expected maintenance costs of *2-out-of-3* system in a single inspection cycle obtained according to Eq. (11)

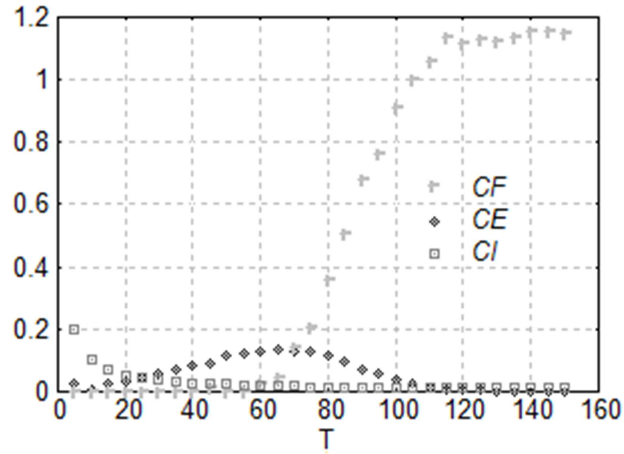

Fig. 3 The simulated costs of a system failure (*CF*), elements' replacement (*CR*) cost and inspection costs (*CI*) in *2-out-of-3* system per a single inspection cycle

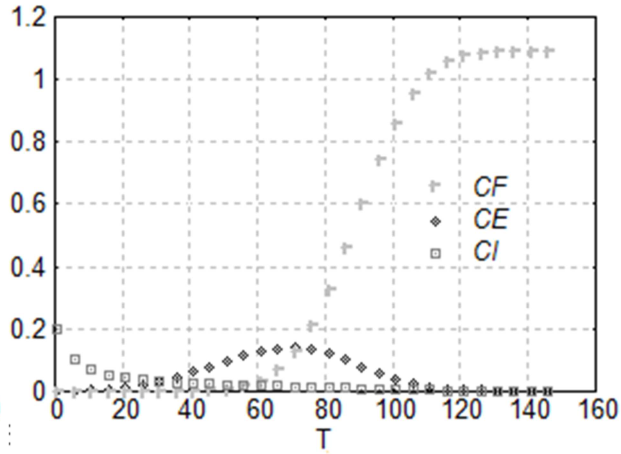

Fig. 4 The costs of a system failure (*CF*), elements' replacement (*CR*) cost and inspection costs (*CI*) in *2-out-of-3* system per a single inspection cycle obtained according to Eq. (11)

The figures 1- 4 confirm the strong convergence of simulation and analytical results of the presented maintenance model. The greatest divergence is observable when an inspection period becomes longer than the mean time to failure of system components ($T > 110$) and is a result of almost zero reliability level of the system. Both the models, simulation and analytical one yield the same results, what may be the foundation to confirm their correctness.

4. SENSITIVITY ANALYSIS

The general maintenance model of *k-out-of-n* systems with delay time, presented in the paper, gives the possibility to use it when parameters of the BIP policy are to be optimized. Even though the minimum of the total maintenance cost for the studied system is placed about $T \approx 20$, the optimum length between inspections should not be set directly on this base. One has to remember of the strong assumption of system components "as good as new" at the beginning of the cycle. That makes model directly usable only for maintenance optimization of systems with a series reliability structure. In the case of *k-out-of-n* systems, when $k \neq n$, a system should be inspected on the base of partial maintenance costs. It seems to be reasonable to inspect system components when:

- the probability of a system failure is low – in order to avoid failure costs,
- the probability of elements replacement is high – to avoid costs of unnecessary inspections.

Thus, the optimum (or at list good) period between two consecutive system inspections may be determined on the base of minimization of the difference of the failure and elements' replacement costs [22]:

(12)

The plot of the *K* value in the function of period between inspections (*T*) for various *k-out-of-n* systems is presented in the figure 5.

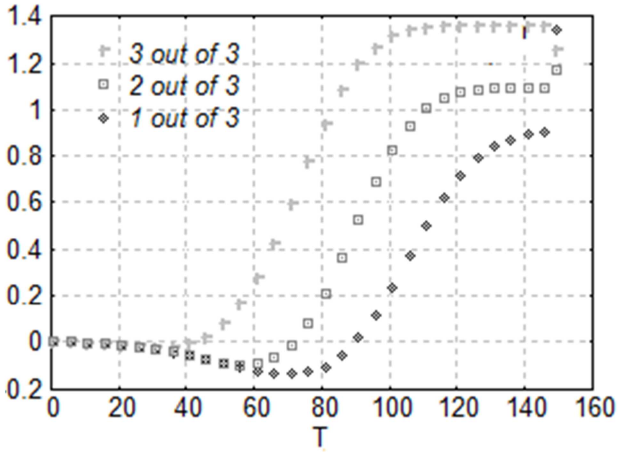

Fig. 5. Function *K* according to Eq. (12)

The results presented in the figure 5 shows different optimum periods between inspections for *k-out-of-n* system, dependently on the *k* value. When system is liable to every component failures (3-*out-of*-3), it should be inspected much more often than systems being more resistant for elements' unreliability ($k < 3$). The presented effect exemplifies the usability of the analytical maintenance model for the general case of system with *k-out-of-n* reliability structure.

4. CONCLUSIONS

In the area of technical systems modeling maintenance modelling using the concept of DT main task is to optimize the length of time between successive inspection actions performance. In the presented article, this problem is solved by developing an analytical model of the expected maintenance cost of technical objects operating in the three main structures reliability (*k-out-of-n*, series and parallel ones). The authors are continuing their research on the mathematical modeling with the use of DT approach for technical systems presented e.g. in [20, 22]. On the other hand, the work continues considerations associated with modeling the maintenance of multi-unit technical systems, presented in [21, 23, 24, 25, 39, 40].

In the next step, authors plan to develop DT model with assumptions being fitted to the real-systems performance (e.g. imperfect maintenance case). The main research efforts are to define some rules how to choose a PM policy from an engineering point of view.

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