Probabilistic Assessment of Composite Plate Failure Behavior under Specific Mechanical Stresses

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Abstract: This research focuses on determination of composite materials reliability and probabilistic assessment of their failure models. The principal task is to determine the probability distribution function for the composite behaviour in order to explain scatter and size effect and to describe composite reliability. A model for the statistical failure of composite materials is presented. As the first step of reliability evaluation, it is essential to understand the candidate failures modes of composite materials and their influence on structural performance. Failure mode and effect analysis (FMEA) is conducted. Based on the FMEA results, failure of a lamina is the main cause of a composite laminate failure. By considering only the failure of lamina, reliability analysis is done by utilizing the Monte Carlo simulation. Also a process is proposed to evaluate the reliability of composite structures. A composite structure of $[0_2/\pm45/90]_4$ graphite-fibre/epoxy-matrix is selected as the case study for the methodology presentation. These result analysis concludes that the Weibull distribution is fitted with enough confidence to represent composite behaviour. In addition to sample size which affects directly accuracy of evaluated reliability, the input variance magnitude is another factor that plays an important role in uncertainty of analysis and converging the results.

Keywords: Composite Materials, Reliability, Monte Carlo Simulation, Wiebull Distribution, Uncertainty

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1. INTRODUCTION

Composites are an important engineering material in construction of automobile, mechanical, space and marine structures in recent years. It resulted in a significant increase in payload, weight reduction, speed, manoeuvrability and durability of products using these materials. In pursuing these achievements, the reliability analysis has thus become an important topic of research. There is considerable statistical variation in mechanical and material properties of composites. Despite years of extensive research around the world, a complete and validated methodology has not yet been fully achieved for predicting the behaviour of composite structures including the effects of damage. This is largely due to the complex nature, so that for any composite structure the performance and the development of damage leading to failure are dependent on a range of parameters including the geometry, material, lay-up, loading conditions, load history and failure modes.

Traditional design methods use global safety factors to take into account the uncertainties in manufacturing, loads, materials properties. Their values have been established after many years of experiments and calibration by judgment, but they are not suited to new materials with particular features. Over the years, a range of stochastic analysis methods have been developed to account for the uncertainties at different scales. Researchers have modelled uncertainty at the micro-scale, as well as macro-scale [1].

Reliability techniques have been in developing since the 1920’s. Cassenti [2] furthered deterministic methods by developing the probabilistic static failure analysis procedure of unidirectional laminated composite structures. Kam [3] predicted the reliability of simply supported angle-ply and cantilever symmetric laminated plates subject to large deflections within the context of first-ply-failure and also developed an analysis procedure for clamped symmetric laminated plates subjected to central point loads based on the first-ply-failure analysis. Chen et al. [4] investigated the reliability of composite tophat stiffened plates for ship hulls providing a rapid analysis for hull girders. Whiteside et al. [5]

showed the effects of using a stochastic failure envelope on uni-directionally stiffened carbon/epoxy composites. Eamon and Rais-Rohani [6] performed a reliability analysis on a full composite boat hull as part of a sizing optimization. An excellent summary and comparison of different reliability studies is given by Sutherland and Guedes Sorares [7]. Also Sorares provides a review of different formulations that have been used to assess the reliability of laminates under plane stress conditions, assuming that they do not fail by delamination.

In this research, an algorithm was presented for reliability evaluation of composite materials. The demonstrated algorithm includes the past researches features and it is arrayed logically and user friendly. For this aim, a special composite laminate was selected as a case study. Since physics of failure method was used for evaluating the reliability because of lack of failure data, FMEA is utilized for identification of effective failure modes in the composite laminate. A composite structure of $[0_2/\pm45/90]_4$ graphite-fibre/epoxy-matrix is selected as the case study. The structure is a rectangular plate with 1×1 dimension in simply support exposure. Since the problem is faced to variety and uncertainty in material properties as input data, Monte Carlo method was implemented to accomplish the non-deterministic calculations. As results of this study, the reliability variations were presented versus different loads, probability distribution function for composite plate failure and the factors affecting it.

2. COMPOSITE FAILURE CRITERIA

Strength of a laminate depends upon the strength of each individual lamina. Therefore the strength of all lamina and arrangement style of them on each other provides a laminate specifications. Based on failure mechanisms governing in composite materials mechanics, it is more appropriate to consider the composite as a structure rather than as a material. It is vital to have required knowledge about failure mechanisms of composite materials before any analysis. In the following sections, it is explained briefly the main causes/ mechanisms of composite materials failure. Since the main failure mode of a laminate is failure of lamina, only this mode is analysed in FMEA and the simulation. Further details about failure modes of a lamina is collected in FMEA table.

2.1. Ply Failure

Composite materials consist of at least two constituents: a series of purposefully oriented fibers, surrounded by a solid matrix. Typically, the fibers act as load-carrying members while the matrix transfers the load between them while fixing the fibers in the desired orientation and location within the composite. The resulting material is both strong and stiff. Composite materials display a wide variety of failure mechanisms as a result of their complex structure and manufacturing processes, which include fiber failure, matrix cracking, buckling and delamination [8].

2.1.1 Fiber Failure

Fiber failure is one of the simplest failure mechanisms to identify and quantify. It occurs when the loads applied to a composite structure cause fracture in the fibers. Fiber failure in tension occurs due to the accumulation of individual fiber failures within plies, which becomes critical when there are not enough intact fibers remaining to carry the required loads.

Fiber failure in compression occurs due to micro buckling and the formation of kink bands, and though there is still debate over whether these phenomena are separate failure modes, micro buckling is a more global failure mode whilst kinking seems to be initiated by local microstructural defects and is the most common failure feature observed after testing.

2.1.2 Matrix Failure

Matrix cracks are an intralaminar form of damage, and involve cracks or voids between fibers within a single composite layer, or lamina. Matrix failure is a complex phenomenon in laminated composites, in which matrix cracks initiate typically at defects or fiber–matrix interfaces, accumulate throughout the laminate, and coalesce leading to failure across a critical fracture plane. Failure modes of a composite lamina under mechanical loads, has been collected in table 1.The FMEA is limited to identification of the potential failure mode, their effect on the composite structure and the causes/mechanisms for such failure development.

Buckling is a structural phenomenon that occurs in compression or shear, and though not necessarily resulting in failure, the large deformations, bending and loss of structural capacity involved typically promotes other types of damage and leads to structural collapse. Delamination are separations between internal layers of a composite laminate caused by high through-thickness stresses, and cause significant structural damage, particularly in compression [9]. As it is mentioned in Failure effect column of table 1, matrix cracking can be considered as the most crucial failure mode of a composite lamina.

Table 1: FMEA of a composite lamina [8-9]

Component	Potential Failure Mode	Failure Effect(s)	Failure Cause(s)/ Mechanisms
Fiber	fiber fracture	- Stiffness reduction - Performance and payload reduction	-longitudinal tensile - transverse tensile - Longitudinal Compressive - in plane shear
	fiber fracture with pullout		-longitudinal tensile - in plane shear
	Micro buckling of fibers		- longitudinal compressive
Matrix	Matrix cracking	- the applied load, results in a crack and the crack grows till the matrix fracture	- longitudinal compressive - transverse tensile
	Delamination	-Structural life reduction - laminate rupture	- interlaminate stress -longitudinal tensile
Fiber-Matrix	Fiber pullout with fiber–matrix debonding	- stiffness reduction	- Longitudinal Tensile - shear in plane
	Fracture of fiber-matrix interface		- transverse tensile

3. COMPOSITES RELIABILITY ANALYSIS AND MODELLING

The need to incorporate uncertainties in an engineering design has long been recognized. The traditional approach, the so-called ‘‘deterministic design’’, makes use of safety coefficients in order to prevent unpredicted failures due to the variability of the data. As a consequence, it is not possible to quantify the reliability of the structure, defined as the probability that the structure does not experience a failure. On the other side, a relatively new trend, named ‘‘probabilistic design’’, allowing the estimation of the reliability of the design, considers the stochastic variability of the data. The performance is generally evaluated by means of a variable such as the displacement of a point, the maximum stress, etc... Due to many reasons (e.g., unpredictability of future loading conditions, inability to express the material properties accurately, simplifications in the modelling of the behavior of the structure, limitations in the numerical methods, human errors or omissions, etc...),the 100% reliability cannot be guaranteed. However, the design can be conducted in order to raise the reliability up to a chosen level. Totally based on the researches have been done in this field, the reliability evaluation process can be divided into five major following steps which includes the past researches background and it is arrayed logically and user friendly. These steps was explained in each part and assumptions have been qualified.

3.1. Random Variables

The first and the most important step for analyzing the reliability of composite materials, is selection of random variables and their statistical distributions. Design parameters are defined by n-dimensional vector $X=(X_1, X2\ldots X_n)^T$ which its elements are uncertain [10].

Random variables considered in this study are shown in Table 2. According to reference [11], the properties and geometry of materials followed normal distribution and a standard deviation of 5% to 20% is allowed range to be assumed. In this article, it is considered 5% as coefficient of variation. The longitudinal tensile strength according to [12], is a Weibull distribution function. The magnitudes of all parameters was brought in table 2.

Table 2: Strength Data of Composite Material

Random variable	**Parameters of normal distribution**		**Parameters of Weibull distribution**	
	μ	σ	α	B
E_1	142.25	7.11	-	-
E_2	8.69	0.43	-	-
G_{12}	4.38	0.219	-	-
υ_{12}	0.24	0.012	-	-
Y_t	51.70	2.585	-	-
h	4.7	0.235	-	-
X_t	-	-	1507.86	75.39

3.2. Failure Criteria

The development of failure criteria for composite materials has been actively pursued for over 30 years by researchers around the world, and there are enormous number of theories available in the literature. In this study, Tsai–Hill criterion of composite laminate plate's failure was selected. This criterion is used for determination of an orthotropic material failure. An orthotropic material has different mechanical properties in three mutually perpendicular directions denoted as 1, 2, and 3, respectively. Composite materials are considered orthotropic in the principal material coordinate system. The Tsai–Hill criterion is expressed as Eq. (1) for a composite material plane layer element subject to stresses in its principal directions [9]:

$$\left[\frac{\sigma 1}{X1}\right]^2-\left[\left(\frac{\sigma_1}{X_2}\right)\left(\frac{\sigma_2}{X_2}\right)\right]+\left[\frac{\sigma 2}{Y}\right]^2+\left[\frac{\tau 12}{S}\right]^2<1 \tag{1}$$

In the Eq. (1), X_1, X_2 and Y obtain one of X_C, X_T, Y_C and Y_T depending upon compressive or tensile magnitudes of σ. The next step is to utilize this information in a reliability model. Unfortunately, principal stresses calculation methods and the Tsai–Hill criterion lead to a very complex expression to compute the probability of failure analytically. Fig.1 expresses the algorithm of calculations utilized in composite materials mechanics. All the steps illustrated in Fig.1 is essential to be performed and as the results of that algorithm, it can be accessible the input parameters of Eq. (1). Since the problem is faced to large amount of data, it is better to utilize a computer program to analyze them. For this reason, MATLAB is selected and the algorithm is applied in it.

3.3. Limit State Function

Limit State Function (LSF) is used to model the reliability. This function is defined by failure scenario of the material and according to the definitions [10], negative magnitudes of this function represents failure. In another word, it is represented as the differences of challenge and strength of material and is defined as Eq. (2). Fig.2 shows the separation of safe region from unsafe region:

$$G(R, S) = G(X_1, X_2 \ldots X_n) = R-S \qquad (2)$$

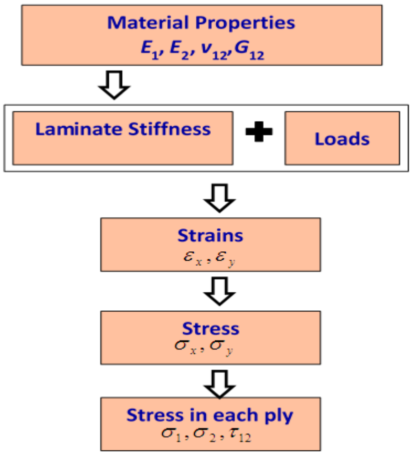

Figure.1: Determine the stress and strain

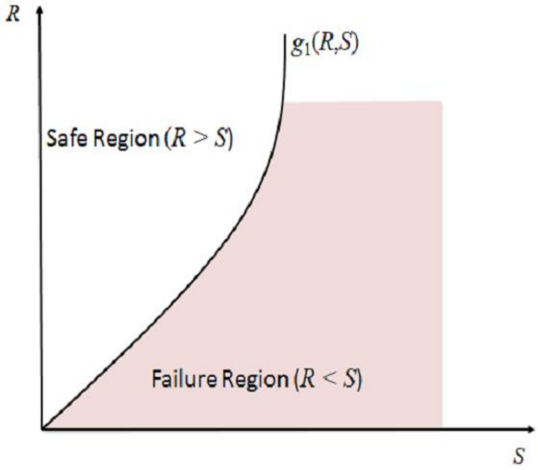

Figure.2:The separation of safe region from failure region by LSF [10]

In this study the limit state function is represented as differences of stress and stength of structure and using Tsai-Hill failure critera. The LSF was defined like Eq. (3):

$$G=1-\left[\frac{\sigma 1}{X1}\right]^2-\left[\left(\frac{\sigma_1}{X_2}\right)\left(\frac{\sigma_2}{X_2}\right)\right]+\left[\frac{\sigma 2}{Y}\right]^2+\left[\frac{\tau 12}{S}\right]^2 \qquad (3)$$

As mentioned before, when G<0, the structure is considered failure.

3.4. Determine the Extent of Structural Strength

From the strength limit point of view, composite laminate failure is classified in two major categories:
1- Last Ply Failure: which is caused by crack in matrix.
2- First Ply failure which is caused by delamination, crack in matrix and fiber failure.
In this study the First ply failure approach was used because of simplicity in Safe Mode determination and more prudence in reliability.

3.5. Reliability Evaluation

The aim of structural reliability analysis is to get the probability of structural failure, while the failure state is denoted by the limit state function. In structural reliability analysis, reliability is defined as a multidimensional nonlinear integral. In the structural reliability analysis, the reliability is defined as:

$$P_f = prob[G(X) \leq 0] = \int_{G(X)\leq 0} f(X)dX \qquad (4)$$

Where G is a performance function or the limit state function, X a vector consisted of the random variables, and f(x) the joint probability density function (PDF) of the random variables X.

Direct evaluation of such an integral is unfeasible or even impossible in most cases. Therefore, some approximation or simulation methods for probabilistic uncertainty analysis have been developed. A direct way to compute this probability of failure is by Monte Carlo simulation. For this particular study, Monte Carlo simulation is preferable to first and second order reliability methods since non-linear complex behavior does not complicate the basic procedure.

Monte Carlo simulation method is sampling procedures for estimating the probability of failure of a component or system. The basic random variables are randomly generated and then inserted a fraction of the overall conditions that lead to failure, as they are considered likely to fail, production and substitution of random variables is in the algorithm illustrated in Fig.3.
Using Monte Carlo simulation, the approximate value of reliability is achieved by the following equation:

$$\hat{R} = \frac{N_S}{N_T} \qquad (5)$$

Where N_s is Total number of successful iterations, and N_T is number of sample size. The algorithm which is illustrated in Fig.3 is related to random variable generation. The random variables are provided for Monte Carlo Method as input data which is brought in table 2. It is essential to use an appropriate sample size in random variable generation in the illustrated algorithm.

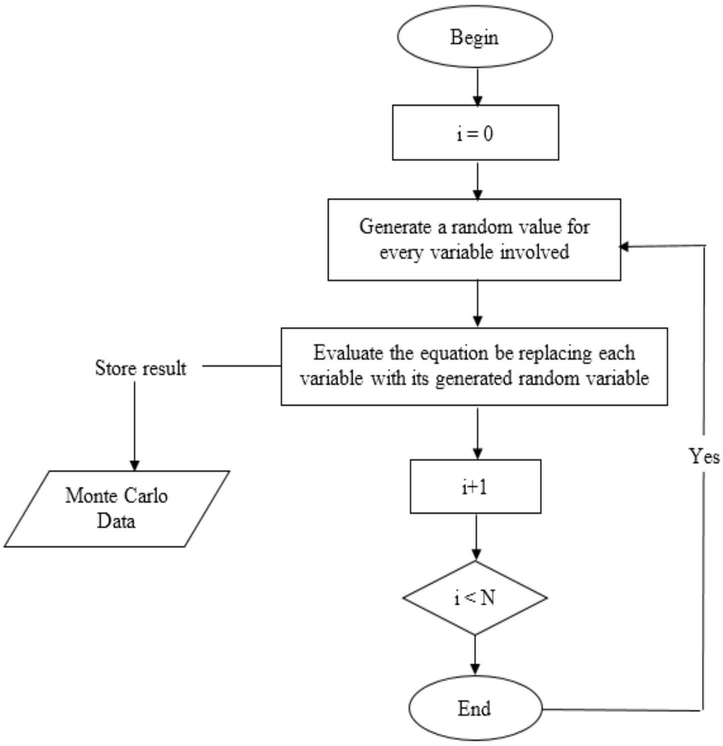

Figure 3: Random variable generation algorithm

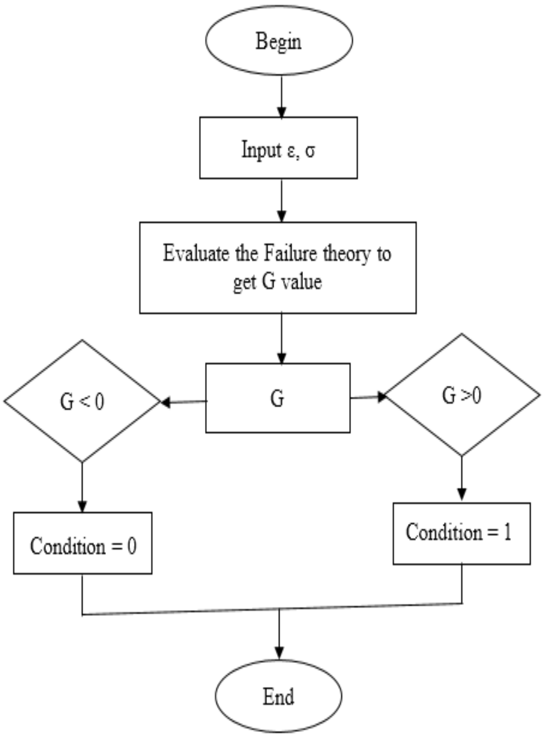

Figure 4: Determination of LSF measure

4. RESULTS

The probability of failure of the plate assumed as a weakest-link system, was calculated by Monte Carlo simulation. The results indicate that p is almost equal to the probability of failure of first layer. By implementation of the presented algorithms illustrated in Fig.3,4, the results is derived including reliability evaluations and fitted failure probability distributions. The sample of 10^6 iteration is determined adequate as the source of input data for MCM. The simulation is performed in MATLAB. Boundary conditions are assumed as simply support and it is considered tensile loads in X and Y direction that are increasingly changed by the fixed step of 200 KN in each loading condition. It was obtained that the model is converged after almost 10^6 iterations. The convergence of the simulation model is illustrated in Fig.5, under [F_x=1500, F_y=1000] KN loading condition.

Utilizing a computer program in MATLAB lead to the conclusion that the 2P-Weibull distribution is the best alternative to describe the failure behavior of the laminate. Easyfit program is used for benchmarking and the results evident the postulate. In this direction, shape and scale parameters of Weibull distribution has been estimated. They are in good agreement as illustrated in Fig.8. The estimated Weibull parameters including the scale and shape parameters for failure probability distribution of composite laminate was collected in table 3. The parameter α represents the reliability. If α is considered as time, when system reaches that time, the probability of failure would be 63.5 percent. In this case study, it is factor of stress as shown in table 3, while declining the load, stress dropped down and consequently the scale parameter plummeted. Also there is a soft increase in β values. Based on bathtub curve can be represented that the increment in load would result in deterioration of the structure.

Table 3: Evaluated reliabilities and Weibull parameters

Condition	Load	Reliability	α	β
1	Fx=800 , Fy=400	≈1	10.2	0.39
2	Fx=1000 , Fy=500	0.99	9.9	0.54
3	Fx=1300 , Fy=800	0.9612	9.2	0.97
4	Fx=1400 , Fy=900	0.8211	9.01	1.21
5	Fx=1500 , Fy=1000	0.5314	8.06	1.28
6	Fx=1800 , Fy=1300	0.011	8.08	1.81

As a result of the simulation, reliability measures was obtained for different loading conditions and collected in Table 3. This is illustrated in Fig.6 where the variation is demonstrated for reliability under each loading condition. The results reveals that the trend is downward gradually following with a sudden decline after the 3rd loading condition. It is justifiable with β measures. There is a jump in β from condition 3 to 4 as illustrated in table 3. In the 4th condition, shape parameter is greater than 1 declaring that it is in deterioration. Therefore the laminate is faced to an impressive reduction in reliability after the 3rd condition.

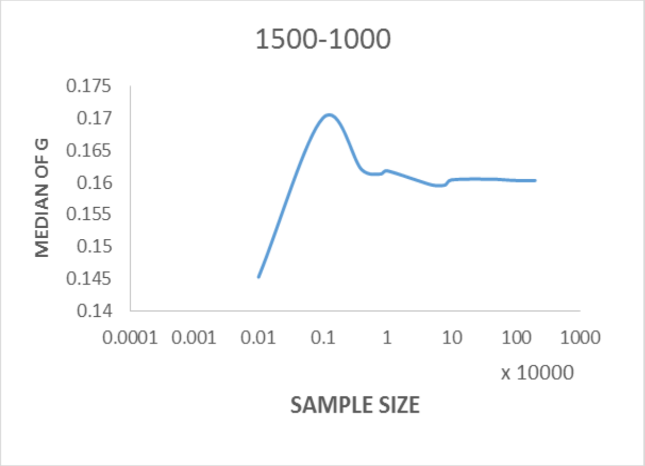

Figure 5: convergence of the simulation under a loading condition

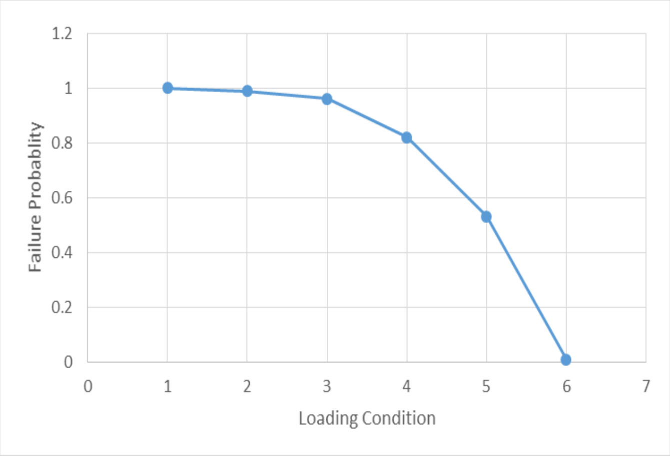

Figure 6: Reliability gradients versus loading conditions

Fig.7 illustrates another concept of failure. Incensement in load, results in incensement in α and the probability of failure which is the area under each probability distribution function diagram is getting large and larger. Furthermore the diagrams by load increment, are driven to left side and according to Weibull distribution concepts, the occurrence of failure has more probability.

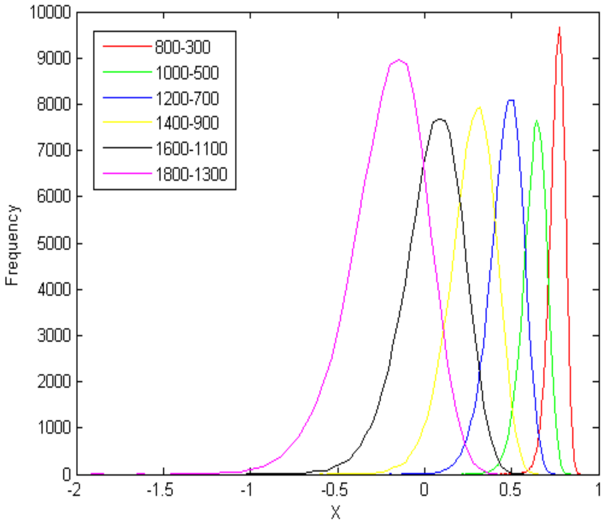

Figure 7: MATLAB output for different loading condition

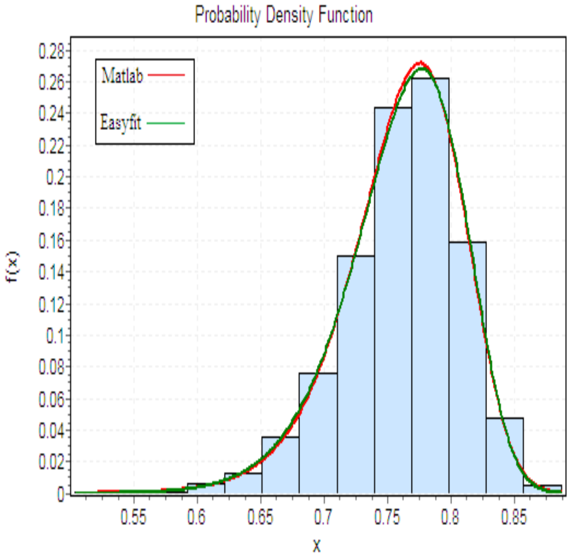

Figure 8: Benchmarking of MATLAB result with Easyfit

Uncertainties always play an important role in reliability assessments. As mentioned before, the standard deviation of input data varies based on various factors such as production process, material deficiencies and so on. In this part, it is assumed that the standard deviation can change almost from 5 to 7.5 percent of median magnitudes. By analyzing the data again, the graph shown in the Fig.9 is resulted. This figure gives good information about the evaluated reliability magnitudes and the probability distribution of composite plate failure. By raising the variance magnitudes of strength factors, the uncertainty trend in data is obviously upwards. This jump is sharper in upper loads. As presented in Fig.9, the last load condition [F_x=1800, F_y=1600] diagram shot up dramatically by contrast of other diagrams and it is while the lower loads diagrams rose gradually at slower pace. This finding, indicates that more uncertainty in input strength data caused by coefficient of variation magnitudes, results in more uncertainty in reliability evaluation.

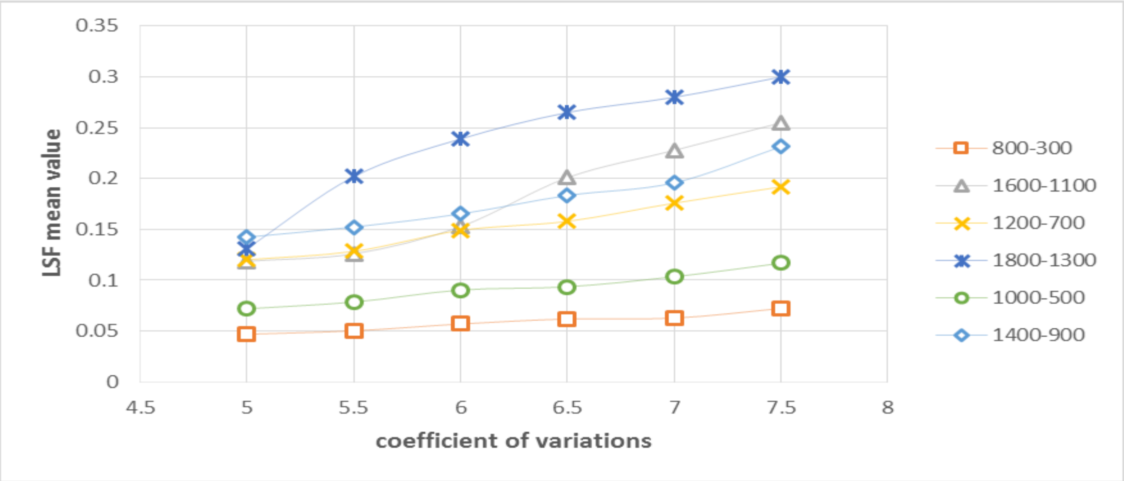

Figure 9: Strength input data Coefficient of variations effect on LSF values

5. CONCLUSION

Since there is uncertainties in composite material strength data, it is essential to utilize the probabilistic assessment in analyzing their specifications. In this paper, an algorithm is presented to evaluate the reliability of composite materials under uncertainty. A composite structure of $[0_2/\pm45/90]_4$ graphite-fiber/epoxy-matrix was selected as the case study for the methodology presentation. These result analysis led to conclusion that the Weibull distribution is fitted with enough confidence to represent composite plate behaviour. In addition to sample size which affects directly accuracy of evaluated reliability, the input variance magnitude is another factor that plays an important role in uncertainty of analysis and converging the results.

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